On the Hadamard's Inverse Mapping Theorem for Continuously Gâteaux Differentiable Mapping

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Abstract

It is well-known that the global invertibility of the derivative of a mapping is not a sufficient condition for the global invertibility of the mapping itself except in the case $f : \mathbb{R} \to \mathbb{R}$. The classic Hadamard's theorem states that for a Banach space X and for a mapping $f : X \to X$ with everywhere continuous Fréchet derivative $f'(x)$, which is invertible everywhere and globally bounded, this is $\exists M$ s.th. $\forall x \in X$ we have $\|[f'(x)]^{-1}\| \leq M$, the mapping f has a smooth inverse mapping g, this is $f(g(x)) = g(f(x)) = x \ \forall x \in X$.

However, Fréchet differentiability is a stronger property of a mapping compared to the Gâteaux differentiability. Here we try to derive a result on the invertibility of a continuously Gâteaux differentiable mapping $f : X \to X$. In our case the assumptions are global boundedness for the inverse of the Gâteaux derivative and we state that for every compact $K \subseteq X$ one can find a right inverse norm-to-norm continuous mapping g such that $f(g(x)) = x \ \forall x \in K$.